**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Soln:- here , *μ=45, σ=8* ,time is 1 hr 60 – 10 =50 : the time left to complete the work

Probability of the service manager cannot meet his commitment

=p(x>50) = 1- pro(x<=50)

X= time taken to the work

Z(standard normal variable) =[(x -*μ*) /*σ* ]

=p(x<=50)=p(z<=(x-45)/8)=p(z<=(50-45)/8)

=p(z<=0.625)=0.7323

=72.23%

So, (100-73.23) = 26.73%

= 0.2676

(i.e: the answer is B)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Soln:- a) *μ* = 38 , *σ* =6

Probability of the employees >44 =pr(x>44)

= 1- pr(x<=44)

Z(standard normal variable) =[(x -*μ*) /*σ* ]

= X-38/6

=pr(z<=(44-38/6))

=pr(z<=1)

=0.8413 =84.13 % , 100 – 84.13 =15.866

Probability of employees between 38 & 44 =pr(x<=44) – pr(x>=38)

Pr(x>=38)= pr(z>=(38-38)/6) = 0.5

Then, pr(x<=44) – pr(x>=38) =0.8413 – 0.5 = 34.134%

The statement is true.

B.) Probability of the employees less than age of 30 = pr(x<30)

Z=[(x -*μ*) /*σ* ]

Pr(x<=30)= pr(z<=(30-38)/6) =pr(z<= -1.333)=9.12% = 0.912

The number of employees wit probability of 0.0912 of them being under age of 30

= (0.0912 x 400) =36.48or 36 employees

So the statement B is also true……

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are (i.i.d) and n is Large. The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are (iid) normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Soln:- Here p(a<x<b)=0.99

Mean = 100

Sd = 20

Here we need to excluded area of 0.005 in each of the left and right tails. Then we want to find the 0.5 th and the 99.5th percentiles z score values..

Z value at 0.5

Z(0.5)=-2.576

Z value at 99.5

Z(99.5) =2.576

Z= (x-100)/20 = <x = 20 z +100

a=-(20\*2.576)+100 =48.5

b = (20\*2.576)+100 =151.5

So the answer is D i.e 48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Soln:- Total profit =profit 1 + profit 2

Profit 1 = N(5,32) , profit 2 =N(7,42)

P1 +P2 =N(5,32) +N(7,42) =(12,74)

Then, [(p-12)/√74] = 0.01

Pr(p<p)= 95%

Pr=[(p-12)/√74] < [(p-12)/√74] =0.95

[(p-12)/√74] ~N(0,1)

P= 12+1.6448 √74=13.415//

P =13.415 = (Pr45)(13.414)

=rs 603.68//

(B) Pr(P<8)= 5%

Pr < [(p-12)/√74]< [(q-12)/√74]=0.05

Then , [(p-12)/√74] ~ N(0,1)

Here x is cde

[(p-12)/√74] =0.05

[(p-12)/√74]= -1.64484

P= 12-1.64484(√74)=10.585

Thus p=approximately 10.585=rs45(10.585)

= rs 476.33

(C)

Probability as 1 st division making less pr(profit 1<0)

Profit 1 ~ N (5,32) =(profit 1 -5)/ √ 32

= ~ N (0,1)

Pr(profit 1 -5)/ √ 32 < -0.88388= 0.18838//